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# Investigation of photonic band gap properties of one-dimensional magnetized plasma spherical photonic crystals

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#### ABSTRACT

Utilizing the transfer matrix method (TMM), the magnetic and electric field equations of Maxwell's system of equations in the spherical coordinate system are constructed. By combining the solution of the spherical Bessel equation to theoretically treat the magnetic and electric fields of a one-dimensional (1-D) magnetized plasma spherical photonic crystals (MPSPCs), detailed expressions for the transmission properties and dispersion relations of 1-D MPSPCs are obtained. The structure of 1-D MPSPCs is also designed, and the effects of the cyclotron frequency of plasma, incidence angle, collision frequency, plasma frequency, structure, dissipation factor and plasma thickness on the photonic band gaps (PBGs) features are discussed. With the adjustment of these parameters, the impact of the collision frequency on the transmission properties is extraordinarily insignificant. The plasma frequency and the cyclotron frequency of plasma have guite similar effects on the PBGs. The variation of incidence angle and plasma frequency thickness affects not only the position of PBGs but also the amount of them. The difference in structure and the change in dissipation factor have no effect on the distribution of PBGs and only a slight effect on the amplitude of the transmission spectra.

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#### **KEYWORDS**

Spherical photonic crystals; magnetized plasma material; photonic band gaps; dispersion relation; transfer matrix method

#### 1. Introduction

Since Yablonovitch [1] and John [2] proposed the concept of photonic crystals (PCs) in 1987, PCs have been the focus of research by scholars in the world. At certain frequencies, electromagnetic waves cannot propagate in PCs, forming photonic band gaps (PBGs) [3,4]. PCs use periodic dielectric structures to modulate electromagnetic waves, and the PBGs formed to offer the possibility of suppressing spontaneous radiation [5]. The discovery of PBGs has led scholars in the fields of photonics [6], electronics [7], and materials physics [8] to make PCs a focus of research, which has brought numerous new applications [9,10].

The one-dimensional (1-D) PCs are of high interest due to their easy fabrication and simple structure. The transmission properties of 1-D PCs can be investigated using the transfer matrix method (TMM). The application of 1-D PCs in the fields of metamaterials [11–13] and superconductors [14–17] have made remarkable development, and their transmittance,

#### 2 😉 T.-Q. ZHU ET AL.

reflectance, and dispersion have been studied intensively. Meanwhile, a multitudinous variety of 1-D PCs-based sensors have been designed in combination with their transmission properties for a wide range of applications such as biosensing [18–21], temperature sensing [22], and physical quantity measurement [23,24]. There are plenty of different forms of electromagnetic waves. With continuous research, researchers have gradually shifted the focus of research from plane waves to cylindrical waves, and 1-D cylindrical photonic crystals (CPCs) have gradually become a research hotspot for scholars at home and abroad [25,26]. Similar to 1-D planar photonic crystals (PPCs), 1-D CPCs can also be processed using the TMM [27–30]. The theoretical development of 1-D CPCs has significant implications in the field of optical fibers [31] and waveguides [32]. In addition to cylindrical waves, spherical waves are also an essential form of electromagnetic waves. The superior optical properties of 1-D CPCs have led scholars to wonder whether symmetrically strong 1-D spherical photonic crystals (SPCs) have more powerful transmission properties. Laviada et al. simulated a new set of spherical waves and used a much smaller number of waves than the original to model the field emitted from an element [33]. In 2016, Wendel and his colleagues solved the problem of the propagation of arbitrary incident waves with the elimination of the prefactor of the radial spherical Bessel function [34]. The above research has refined the theory of spherical waves and laid the foundation for the study of 1-D SPCs. Nowadays, 1-D SPCs are widely used in biological [35], electronic communication [36], and sensor fields [37] and have tremendous research and development prospects.

The concept of 1-D magnetized plasma photonic crystals (MPPCs) was first proposed by the Japanese scholars Hojo and Mase [38]. In this study, they theoretically deduced the dispersion relation of 1-D MPPCs and performed a detailed analysis. Laxmi and Parmanand derived the calculations of the finite and infinite period structure of the plasma [39]. The properties of the PBGs are discussed in terms of plasma density, plasma width, and unit cell number. Numerical simulations of non-magnetized plasma and dielectric materials using the finite difference time domain method were carried out by Liu and his coworkers [40]. Some of the parameters in 1-D MPPCs can be changed, which makes them tunable [41,42]. This feature makes 1-D MPPCs of considerable interest for the design of space filters [43], tunable ultrawideband absorbers [44], and multichannel filters [45]. The non-reciprocal properties of 1-D plasma CPCs have been investigated by Wang et al. [46]. Although there are numerous studies on plasma and 1-D PCs, the system of PCs is not well established and the main research of 1-D SPCs is still at the stage of theory and design of simple devices. In 2016, Wendel and his team studied the unfolding of vector spherical waves in electromagnetic fields and obtained a complete set of radially independent amplitudes of vector spherical wave functions [47]. What's more, 1-D SPCs are momentous in optical displays [48], millimeter wave communication [49], nanomaterials [50], and other fields. It is no exaggeration to say that 1-D SPCs have well-defined geometrical properties and adapt to a wide range of physical scenarios, there is no exploration keeping a watchful eye on the combination of 1-D SPCs and plasma to construct adaptable and multi-functional 1-D magnetized plasma spherical photonic crystals (MPSPCs).

In this paper, a systematic theoretical analysis of the transmission properties of 1-D SPCs with magnetized plasma using TMM is carried out, and detailed expressions for the reflectance, transmittance, absorptance and dispersion relations are acquired. Due to the magneto-optical effect, the study is concentrated on the case of TM waves. A 1-D MPSPCs structure was also constructed, and the effects of the cyclotron frequency of plasma,



**Figure 1.** The structure chart of 1-D MPSPCs with periodic arrangement of the general dielectric K, the general dielectric L and the plasma.

incidence angle, collision frequency, plasma frequency, structure, dissipation factor, and plasma thickness on the PBGs performance of 1-D MPSPCs are investigated. It is revealed that 1-D MPSPCs not only have superior forbidden band properties but also have tunability and favorable absorption properties. Their superior transmission properties provide new insights into the design of tunable electromagnetic devices and reflectors. It is worth mentioning that this paper only focuses on the theoretical investigation, and how to realize and verify the given 1-D MPSPCs or obtained results in the experiment is beyond scope of this paper.

## 2. Simulation model and formulation

The structure of 1-D MPSPCs is shown in Figure 1, which consists of the general dielectric K, the general dielectric L and the plasma P arranged periodically according to the  $(PKL)^N$  form, where N represents the number of periods. Normalize d with d = 1, initial radius  $r_0$  is taken as 20d, thickness  $d_A$  of general dielectric K is 0.2d, thickness  $d_B$  of general dielectric L is 0.35d, and thickness  $d_P$  of plasma P is 0.45d. The effective refractive index of the general dielectric L is  $n_K = 2.8$ , the effective refractive index of the general dielectric L is  $n_L = 2.1$ , and the effective magnetic permeability of plasma P will be described in detail below.

Due to the special geometric properties of the sphere, inspired by the idea of the microelement method and the definition of the incidence angle under the cylindrical photonic crystal [27], the TE waves and TM waves are defined by taking the profile of the spherical wave and 1-D MPSPCs parallel to the wave propagation direction, as shown in Figure 2. For this plane, the electric field **E** is perpendicular to the plane, the magnetic field **H** is parallel to the plane, and the wave vector **k** indicates the propagation direction. Therefore, the electric field **E** takes the form of **E** = (0,  $E_{\theta}$ , 0) in the TE waves and the magnetic field **H** takes the form of **H** = ( $H_r$ , 0,  $H_{\varphi}$ ). By analogy, in the TM waves, the electric field *E* is denoted as **E** = ( $E_r$ , 0,  $E_{\varphi}$ ) and the magnetic field *H* is denoted as **H** = (0,  $H_{\theta}$ , 0). 1-D MPSPCs and the spherical waves demonstrably have an intersection point. From the intersection point, the tangents to 1-D MPSPCs and the spherical waves are made respectively, and the angle between the two tangents is the angle of incidence, as shown in Figure 2. The electromagnetic wave propagates from the *yoz* plane at an angle of incidence  $\alpha$ . It is prescribed that



**Figure 2.** The main image of 1-D MPSPCs showing in detail the angle of incidence  $\alpha$ , electromagnetic direction and magnetic field direction.

under the TE polarization the wave vector  $\mathbf{k}$  is always perpendicular to the magnetic field  $\mathbf{H}$ . Equally, under TM polarization the electric field  $\mathbf{E}$  invariably remains perpendicular to the wave vector  $\mathbf{k}$ .

The system of Maxwell's equations with time using plasma is expressed as:

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}$$
<sup>(2)</sup>

$$\frac{d\mathbf{J}}{dt} + \mathbf{v}_c \mathbf{J} = \varepsilon_0 \omega_p^2 \mathbf{E} + \boldsymbol{\omega_c} \times \mathbf{J}$$
(3)

where **J** is a vector quantity, called polarization current density, which can be decomposed along the three directions r,  $\theta$ , and  $\psi$ .  $v_c$  is the collision frequency and  $\omega_p$  is the plasma frequency.  $\omega_c$  is the cyclotron frequency of plasma, denoted as  $eB_0/me_{\theta}$ . In the initial state, define normalized frequency  $\omega_0 = 2\pi c/d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\omega_c = 0$ .

Decomposing Equation (3) in the three directions of r,  $\theta$ , and  $\psi$ , the following is obtained.

$$(-i\omega + v_c)J_r = \varepsilon_0 \omega_p^2 E_r + \omega_c J_\varphi \tag{4}$$

$$(-i\omega + v_c)J_{\theta} = \varepsilon_0 \omega_p^2 E_{\theta}$$
<sup>(5)</sup>

$$(-i\omega + v_c)J_{\varphi} = \varepsilon_0 \omega_p^2 E_{\varphi} - \omega_c J_r \tag{6}$$

In consideration of the relationship between **J** and **E**, **J** is expressed in the following form:

$$\begin{pmatrix} J_{r} \\ J_{\theta} \\ J_{\varphi} \end{pmatrix} = \varepsilon_{0} \begin{pmatrix} \frac{i\omega_{p}^{2}(\omega + iv_{c})}{(\omega + iv_{c})^{2} - \omega_{c}^{2}} & 0 & -\frac{\omega_{p}^{2}\omega_{c}}{(\omega + iv_{c})^{2} - \omega_{c}^{2}} \\ 0 & \frac{i\omega_{p}^{2}}{\omega + iv_{c}} & 0 \\ \frac{\omega_{p}^{2}\omega_{c}}{(\omega + iv_{c})^{2} - \omega_{c}^{2}} & 0 & \frac{i\omega_{p}^{2}(\omega + iv_{c})}{(\omega + iv_{c})^{2} - \omega_{c}^{2}} \end{pmatrix} \begin{pmatrix} E_{r} \\ E_{\theta} \\ E_{\varphi} \end{pmatrix}$$
(7)

Since Equation (7) gives the relationship between **J** and **E**, Equation (2) can be rewritten as:

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \varepsilon_p \frac{\partial \boldsymbol{E}}{\partial t} \tag{8}$$

where

$$\boldsymbol{\varepsilon}_{\boldsymbol{p}} = \begin{pmatrix} \varepsilon_1 & 0 & i\varepsilon_2 \\ 0 & \varepsilon_3 & 0 \\ -i\varepsilon_2 & 0 & \varepsilon_1 \end{pmatrix}$$
(9)

$$\varepsilon_1 = 1 - \frac{\omega_p^2(\omega + i\mathbf{v}_c)}{\omega[(\omega + i\mathbf{v}_c)^2 - \omega_c^2]}$$
(10)

$$\varepsilon_2 = -\frac{\omega_p^2 \omega_c}{\omega[(\omega + iv_c)^2 - \omega_c^2]}$$
(11)

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega(\omega + iv_c)} \tag{12}$$

Under the TM polarization, the electric and magnetic fields are represented as:

$$\boldsymbol{E} = e^{-i\omega t}(E_r, 0, E_{\varphi}) \tag{13}$$

$$\boldsymbol{H} = \boldsymbol{e}^{-i\omega t}(0, H_{\theta}, 0) \tag{14}$$

Maxwell's system of equations is formulated as follows:

$$\frac{1}{r\sin\theta} \left[ \frac{\partial E_r}{\partial \varphi} - \frac{\partial}{\partial r} (r\sin\theta E_{\varphi}) \right] = -i\omega\mu_0 H_{\theta}$$
(15)

$$\frac{1}{r^2 \sin \theta} \left[ \frac{\partial}{\partial \theta} (r \sin \theta H_{\varphi}) - \frac{\partial}{\partial \varphi} (r H_{\theta}) \right] = i \omega \varepsilon_0 (\varepsilon_1 E_r + i \varepsilon_2 E_{\varphi})$$
(16)

$$\frac{1}{r} \left[ \frac{\partial}{\partial r} (rH_{\theta}) - \frac{\partial H_r}{\partial \theta} \right] = i\omega\varepsilon_0 (-i\varepsilon_2 E_r + \varepsilon_1 E_{\varphi})$$
(17)

Combined with the transmission properties of spherical waves, the derivative of  $\psi$  can be omitted.

$$\frac{1}{r^2 \sin \theta} \left[ -\frac{\partial}{\partial \varphi} (rH_\theta) \right] = i\omega \varepsilon_0 (\varepsilon_1 E_r + i\varepsilon_2 E_\varphi)$$
(18)

$$\frac{1}{r} \left[ \frac{\partial}{\partial r} (rH_{\theta}) \right] = i\omega\varepsilon_0 (-i\varepsilon_2 E_r + \varepsilon_1 E_{\varphi})$$
(19)

6 🔄 T.-Q. ZHU ET AL.

Simplifying Equations (18) and (19) yields:

$$E_{\varphi} = \frac{1}{r\omega\varepsilon_0} \frac{1}{\varepsilon_2^2 - \varepsilon_1^2} \left[ \frac{\varepsilon_2}{r\sin\theta} \frac{\partial(rH_{\theta})}{\partial\varphi} + i\varepsilon_1 \frac{\partial(rH_{\theta})}{\partial r} \right]$$
(20)

$$E_r = \frac{1}{r\omega\varepsilon_0} \frac{1}{\varepsilon_2^2 - \varepsilon_1^2} \left[ \varepsilon_2 \frac{\partial (rH_\theta)}{\partial r} - \frac{i\varepsilon_1}{r\sin\theta} \frac{\partial (rH_\theta)}{\partial \varphi} \right]$$
(21)

Substitute Equations (20) and (21) into Equation (15).

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial H_\theta}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 H_\theta}{\partial \varphi^2} + k^2 H_\theta = 0$$
(22)

Furthermore, here  $k = \omega \cos \alpha \sqrt{\mu_0 \varepsilon_0 \varepsilon_{TM}}$ ,  $\varepsilon_{TM} = (\varepsilon_1^2 - \varepsilon_2^2)/\varepsilon_1$  and  $\alpha$  is the incident angle. In the following, define  $H_{\theta}(x) = V(x)\Psi(\varphi)$ , where x = kr. Readily know the angular part of  $H_{\theta}$  satisfies the following equation.

$$\frac{d^2\Psi}{d\varphi^2} + m^2\varphi = 0 \tag{23}$$

Solving differential Equation (23) yields  $\Psi \sim e^{im\varphi}$ , where *m* is a positive integer, a negative integer, and zero.

Then let,

$$m^2 = l(l+1)\sin^2\theta \tag{24}$$

Utilizing the separation of variables method, the equations on V(x) are derived by combining Equations (22)–(24).

$$x^{2}\frac{d^{2}V}{dx^{2}} + x\frac{dV}{dx} + \left[x^{2} - \left(I + \frac{1}{2}\right)^{2}\right]V(x) = 0$$
(25)

It is effortless to conclude that V(x) fulfills the form of the standard semi-odd order spherical Bessel equation. Therefore, V(x) can be written in the form of a linear combination of spherical Bessel functions.

$$V(x) = Aj_l(x) + Bn_l(x)$$
<sup>(26)</sup>

Both  $j_l(x)$  and  $n_l(x)$  are solutions of the spherical Bessel equation of semi-odd order, and the expressions are shown below.

$$j_{l}(x) = \sqrt{\frac{\pi}{2x}} J_{l+\frac{1}{2}}(x)$$

$$n_{l}(x) = \sqrt{\frac{\pi}{2x}} N_{l+\frac{1}{2}}(x)$$
(27)

For simplicity, U(x) is expressed as:

$$U(x) = \frac{1}{i\omega\varepsilon r}\frac{\partial}{\partial x}(xH_{\theta}) = \frac{1}{i\omega\varepsilon r}\left(H_{\theta} + x\frac{\partial H_{\theta}}{\partial x}\right)$$
(28)

Substitute Equation (26) into Equation (28)

$$U(x) = \frac{1}{i\omega\varepsilon r} [Aj_{l}(x) + Bn_{l}(x) + xAj_{l}'(x) + xBn_{l}'(x)]$$
(29)

Meanwhile,  $j'_{l}(x)$  and  $n'_{l}(x)$  are the first order derivatives of the Bessel functions, and the corresponding equations are indicated as:

$$j_{l}'(x) = \sqrt{\frac{\pi}{2x}} J'_{l+\frac{1}{2}}(x) - \frac{1}{2} \sqrt{\frac{\pi}{2}} x^{-\frac{3}{2}} J_{l+\frac{1}{2}}(x)$$
$$n_{l}'(x) = \sqrt{\frac{\pi}{2x}} N'_{l+\frac{1}{2}}(x) - \frac{1}{2} \sqrt{\frac{\pi}{2}} x^{-\frac{3}{2}} N_{l+\frac{1}{2}}(x)$$
(30)

1-D MPSPCs are formed by periodic arrangements of plasma and general dielectrics. To obtain the transmission properties of electromagnetic waves in the multilayer dielectrics, the vector  $\begin{pmatrix} V(x) \\ U(x) \end{pmatrix}$  is stipulated and the TMM is adopted to relate the homologous vectors at different radiuses.

$$\begin{pmatrix} V(x) \\ U(x) \end{pmatrix} = \mathbf{M} \begin{pmatrix} V(x_0) \\ U(x_0) \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V(x_0) \\ U(x_0) \end{pmatrix}$$
(31)

The electric and magnetic fields at  $r_0$  and r are facile to receive the respective magnitudes, and the work to be performed now is the solution of the elements of the transmission matrix. To make the calculation easier, the method of taking special values is employed. In the first place, let,

$$V(x_0) = 1$$
  
 $U(x_0) = 0$  (32)

From the above derivations, we have:

$$M_{11} = V(x) = Aj_l(x) + Bn_l(x)$$
(33)

$$M_{21} = U(x) = \frac{1}{i\omega\varepsilon r} [Aj_l(x) + Bn_l(x) + xAj_l'(x) + xBn_l'(x)]$$
(34)

Assign values to V(x) and U(x)

$$V(x_0) = Aj_l(x_0) + Bn_l(x_0) = 1$$
  

$$U(x_0) = Aj_l(x_0) + Bn_l(x_0) + x_0Aj_l'(x_0) + x_0Bn_l'(x_0) = 0$$
(35)

Make use of Equation (35), the expressions for A and B can be determined.

$$A = \frac{1}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[n_{l}'(x_{0}) + \frac{n_{l}(x_{0})}{x_{0}}\right]$$
$$B = \frac{1}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[j_{l}'(x_{0}) + \frac{j_{l}(x_{0})}{x_{0}}\right]$$
(36)

8 😧 T.-Q. ZHU ET AL.

where A and B are crucial components of the elementary expressions of the matrix. Combining Equations (33), (34) and (36), the expressions for  $M_{11}$  and  $M_{21}$  are derived.

$$M_{11} = \frac{j_{l}(x)}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[n_{l}'(x_{0}) + \frac{n_{l}(x_{0})}{x_{0}}\right] + \frac{n_{l}(x)}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[j_{l}'(x_{0}) + \frac{j_{l}(x_{0})}{x_{0}}\right]$$
(37)

$$M_{21} = \frac{1}{i\omega\varepsilon r} \left\{ \frac{j_{l}(x)}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[ n_{l}'(x_{0}) + \frac{n_{l}(x_{0})}{x_{0}} \right] \right. \\ \left. + \frac{n_{l}(x)}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[ j_{l}'(x_{0}) + \frac{j_{l}(x_{0})}{x_{0}} \right] \right. \\ \left. + \frac{xj_{l}'(x)}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[ n_{l}'(x_{0}) + \frac{n_{l}(x_{0})}{x_{0}} \right] \right. \\ \left. + \frac{xn_{l}'(x)}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[ j_{l}'(x_{0}) + \frac{j_{l}(x_{0})}{x_{0}} \right] \right\}$$
(38)

Similarly, assigning values to V(x) and U(x),

$$V(x_0) = 0$$
  
 $U(x_0) = 1$  (39)

With Equation (31), it is possible to observe that

$$M_{12} = V(x) = Cj_l(x) + Dn_l(x)$$
(40)

$$M_{22} = U(x) = \frac{1}{i\omega\varepsilon r} [Cj_l(x) + Dn_l(x) + xCj_l'(x) + xDn_l'(x)]$$
(41)

Then we get:

$$V(x_0) = Cj_l(x_0) + Dn_l(x_0) = 0$$
  

$$U(x_0) = \frac{1}{i\omega\varepsilon r_0} [Cj_l(x_0) + Dn_l(x_0) + x_0Cj_l'(x_0) + x_0Dn_l'(x_0)] = 1$$
(42)

Integrating Equation (42), the expressions for C and D are shown below.

$$C = \frac{1}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[\frac{i\omega\varepsilon}{k}n_{l}(x_{0})\right]$$
$$D = \frac{1}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[\frac{i\omega\varepsilon}{k}j_{l}(x_{0})\right]$$
(43)

 $M_{12}$  and  $M_{22}$  are further written as:

$$M_{12} = \frac{j_l(x)}{n_l(x_0)j_l'(x_0) - n'_l(x_0)j_l(x_0)} \left[\frac{i\omega\varepsilon}{k}n_l(x_0)\right] + \frac{n_l(x)}{j_l(x_0)n'_l(x_0) - n_l(x_0)j_l'(x_0)} \left[\frac{i\omega\varepsilon}{k}j_l(x_0)\right]$$
(44)

# WAVES IN RANDOM AND COMPLEX MEDIA 😔 9

$$M_{22} = \frac{1}{i\omega\varepsilon r} \{ \frac{j_{l}(x)}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[ \frac{i\omega\varepsilon}{k} n_{l}(x_{0}) \right] \\ + \frac{n_{l}(x)}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[ \frac{i\omega\varepsilon}{k} j_{l}(x_{0}) \right] \\ + \frac{xj_{l}'(x)}{n_{l}(x_{0})j_{l}'(x_{0}) - n'_{l}(x_{0})j_{l}(x_{0})} \left[ \frac{i\omega\varepsilon}{k} n_{l}(x_{0}) \right] \\ + \frac{xn_{l}'(x)}{j_{l}(x_{0})n'_{l}(x_{0}) - n_{l}(x_{0})j_{l}'(x_{0})} \left[ \frac{i\omega\varepsilon}{k} j_{l}(x_{0}) \right]$$
(45)

At this point, the transmission matrix is derived. For wave propagation, the field can be signified as a sum of two waves in opposite directions, which can be considered as a superposition of incoming and outgoing waves. These two waves are usually represented by two Hankel functions:

$$h_{1}^{(1)}(x) = j_{l}(x) + in_{l}(x)$$

$$h_{1}^{(2)}(x) = j_{l}(x) - in_{l}(x)$$
(46)

The magnetic and electric fields can be expressed separately as:

$$H_{\theta}^{+}(x) = Qh_{l}^{(2)}(x)e^{im\varphi}$$
  

$$H_{\theta}^{-}(x) = Ph_{l}^{(1)}(x)e^{im\varphi}$$
(47)

$$E_{\varphi}^{+}(x) = \frac{Q}{i\omega\varepsilon r} [h_{l}^{(2)}(x) + xh_{l}^{(2)'}(x)]e^{im\varphi}$$
$$E_{\varphi}^{-}(x) = \frac{P}{i\omega\varepsilon r} [h_{l}^{(1)}(x) + xh_{l}^{(1)'}(x)]e^{im\varphi}$$
(48)

From Maxwell's system of equations, the following relationship between the electric field and the magnetic field can be realized.

$$C_{l}^{(2)}(x) = 1 + x \frac{h_{l}^{(2)'}(x)}{h_{l}^{(2)}(x)}$$

$$C_{l}^{(1)}(x) = 1 + x \frac{h_{l}^{(1)'}(x)}{h_{l}^{(1)}(x)}$$
(49)

The output waves are incident to the  $r = r_0$  interface and finally exit from the  $r = r_f$  interface. With the boundary conditions in mind, TMM is selected to string together 1-D SPCs at different radii. For the sake of observation and understanding, here we assume the case where the incidence angle is 0 as an example. The incident spherical wave is defined as 1 and is analogous to the Airy formula in the plane. The outgoing spherical waves can be expressed by the transmission coefficient  $t_d$  and the reflected spherical waves by the reflection coefficient  $r_d$ . The expressions for the electric and magnetic fields on both sides of the dielectric can be with a wet finger obtained by theoretical calculations using TMM, which in turn leads to the expressions for the transmission properties of 1-D MPSPCs [27] (Figure 3).



Figure 3. The schematic diagram of spherical waves propagation at positive incidence, where the surface interface is intentionally drawn as a straight line for ease of illustration.

In 1-D MPSPCs, the evanescent spherical waves are incident at  $r_0$  and exit at  $r_f$ . The reflection coefficient  $r_d$  and transmission coefficient  $t_d$  can be described by the elements in the transmission matrix **M**.

$$\begin{pmatrix} 1+r_d \\ \frac{C_l^{(2)}(x_0)}{i\omega\varepsilon_0 r_0} + \frac{C_l^{(1)}(x_0)}{i\omega\varepsilon_0 r_0} r_d \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} t_d \\ \frac{C_l^{(2)}(x_f)}{i\omega\varepsilon_f r_f} t_d \end{pmatrix}$$
(50)

where,

$$M = M_1 M_2 M_1 \cdots M_1 M_2$$
$$M^{-1} = \begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix}$$
(51)

Substituting Equations (37), (38), (44) and (45) into Equation (50), the reflection coefficient  $r_d$  and transmission coefficient  $t_d$  can be expressed in terms of the elements in the transfer matrix **M**. The specific expression is written as:

$$t_{d} = \frac{C_{l}^{(1)}(x_{0}) - C_{l}^{(2)}(x_{0})}{i\omega\varepsilon_{0}r_{0} \left[M_{11}\frac{C_{l}^{(1)}(x_{0})}{i\omega\varepsilon_{0}r_{0}} - M_{21} + \frac{C_{l}^{(2)}(x_{f})}{i\omega\varepsilon_{f}r_{f}} \cdot \frac{C_{l}^{(1)}(x_{0})}{i\omega\varepsilon_{0}r_{0}}M_{12} - M_{22}\frac{C_{l}^{(2)}(x_{f})}{i\omega\varepsilon_{f}r_{f}}\right]}$$
(52)

$$r_{d} = M_{11} + M_{12} \frac{C_{l}^{(5)}(x_{f})}{i\omega\varepsilon_{f}r_{f}}$$
(53)

Ultimately get,

$$T = |t_d|^2 R = |r_d|^2$$
(54)

$$A = 1 - T - R \tag{55}$$

Integrating the above theoretical treatment, the transmittance and reflectance formulas for 1-D MPSPCs are deduced completely.

10

For the periodic structure in Figure 1, the following equation can be obtained by combining Bloch's theorem.

$$\begin{pmatrix} E_{N+3} \\ H_{N+3} \end{pmatrix} = e^{ikd} \begin{pmatrix} E_N \\ H_N \end{pmatrix}$$
(56)

Similarly, from the TMM, the electric and magnetic fields of the N layer are related to the electric and magnetic fields of the N + 3 layer as follows:

$$\begin{pmatrix} E_N \\ H_N \end{pmatrix} = M_1 M_p M_2 \begin{pmatrix} E_{N+3} \\ H_{N+3} \end{pmatrix}$$
(57)

Combined with the knowledge associated with Bloch's theorem, we are informed that when the transmission mode and frequency are known, the difference between the fields in two cross sections at a distance of one spatial cycle length is a complex constant, and the relationship between the electric and magnetic field distribution under multiple periods can be obtained by investigating the electromagnetic properties in one period. TMM is also exploited to achieve the target of analyzing the electromagnetic wave distribution on both sides of the dielectric. By and large, use Bloch's theorem, as in Equation (56), to simplify the study range from multi-period to single-period, and employ TMM to perform the theoretical analysis, as in Equation (57), which fully takes into account the propagation of electromagnetic waves in different dielectrics, in accordance with the actual scenario.

Synthesizing Equations (55) and (56), the dispersion relation satisfies:

$$\det(\boldsymbol{M_1}\boldsymbol{M_p}\boldsymbol{M_2} - e^{-ikd}) = 0 \tag{58}$$

Let,

$$M_1 M_p M_2 = M_k = \begin{pmatrix} M_{k11} & M_{k12} \\ M_{k21} & M_{k22} \end{pmatrix}$$
(59)

The ultimate dispersion relationship satisfies the following equation.

$$\cos(kd) = \frac{M_{k11} + M_{k22}}{1 + \det M_k} = \frac{M_{k11} + M_{k22}}{2}$$
(60)

#### 3. Discussion

Figure 4 illustrates the reflectance spectra of  $\omega_c$  changing from 0 to 1. The initial parameters are set as follows:  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ . And we can plainly observe that a new PBG is generated between  $2.3\omega_0$  and  $2.5\omega_0$  as  $\omega_c$  increases. Since the low reflectivity region accounts for a diminutive percentage, the following reflectance spectra and dispersion curves of specific  $\omega_c$  are combined to analyze the transformation law. For the sake of representing PBGs more visually, it is decided to symbolize the location of PBGs with gray shading regions.

From Figures 5 and 6, it can become conscious that when  $\omega_c = 0$ , apparent PBGs can be observed at  $\omega/\omega_0 = 2.02$ , 2.60, and 2.87. At this stage, the external magnetic field is not available and the magneto-optical Voigt effect is not manifested in the plasma layer. If  $\omega_c$  is  $0.8\omega_p$ , the position of PBG originally located at  $\omega/\omega_0 = 2.02$  does not move significantly, but



**Figure 4.** The reflection mapping in the case of  $\omega_c$  variation, and the angle coordinate is  $\omega_c/\omega_p$  with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ .

the bandwidth rises from  $0.12\omega_0$  to  $0.14\omega_0$ . Homoplastically, the frequency range of the PBG initially located at  $\omega/\omega_0 = 2.87$  is expanded from  $2.87-2.94\omega/\omega_0$  to  $2.87-2.96\omega/\omega_0$ . Compared with the PBG located at  $2.60\omega_0$  at  $\omega_c = 0$ , the frequency coverage area of the PBG in the case of  $\omega_c = 0.8$  is lessened. Further enhancing  $\omega_c$  to  $\omega_p$ , it can be noted that the first PBG moves from  $2.02\omega_0$  to  $2.04\omega_0$ , showing a holistic trend toward the high-frequency region. And the bandwidths of the PBGs in this condition acquire elevated and the maximum value enlarges to  $0.16\omega_0$ . On the side, it is evident from the dispersion curve that when  $\omega_c = 0.8\omega_p$ , a PBG emerges at  $\omega/\omega_0 = 2.32$ , which does not appear at  $\omega_c = 0$ . And as  $\omega_c$  ulteriorly increases to  $\omega_p$ , this PBG moves to high frequencies, accompanied by an expansion in bandwidth, the alteration of  $\omega_c$  provides a fire-new idea for the adjustment of the position and frequency range. For the PBGs having narrow bandwidths, the effect of  $\omega_c$  on them is dubious, which presumably gives rise to the expansion of the forbidden bands and the SBG moves.

Figures 7–9 are utilized below to explore the impact of  $\alpha$  variation on the reflection spectra. Define initial parameters  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ , and  $v_c = 0.0001\omega_p$ . It can be undemanding concluded from Figure 8 that the bandwidths of PBGs tend to enlarge and the number decreases as  $\alpha$  grows. This property is extremely similar to that of 1-D PPCs, and it follows that the large incidence angles are one of the necessary conditions for the formation of the wide PBGs. The next part provides a particular analysis of  $\alpha$ .

According to Figure 8(b), it can be figured out that the PBGs can be observed with a wet finger at  $\omega/\omega_0 = 1.97$ , 2.29, and 2.97 for  $\alpha = 30^\circ$ . The electromagnetic waves are unable to pass through the frequency range of  $1.97\omega_0-2.11\omega_0$ ,  $2.29\omega_0-2.46\omega_0$ , and  $2.97\omega_0-3.05\omega_0$ , compared to Figure 7(a), the most prominent difference is the location of the PBGs. For Figure 7(c), if  $\alpha = 45^\circ$ , the PBGs are located at  $2.02\omega_0$ ,  $2.39\omega_0$ , and  $2.80\omega_0$  and are equipped with wide bandwidths. All the forbidden bandwidths are more than  $0.1\omega_0$ , and the maximum frequency region reaches  $0.18\omega_0$ . In the case of  $\alpha = 60^\circ$ , only one PBG appears because of the large angle of incidence at this point. This PBG lies in  $\omega/\omega_0 = 2.78-3.02$  and



**Figure 5.** The influences of  $\omega_c$  on the reflection spectra in case of  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ , (a)  $\omega_c = 0$ , (b)  $\omega_c = 0.8\omega_p$ , and (c)  $\omega_c = 1.2\omega_p$ .

the forbidden frequency interval is much larger than the conditions of Figure 8(a-c). Meanwhile, comparing the results in Figure 8 with Figure 9, it is detected that the positions and bandwidths of the PBGs match perfectly, further proving the correctness of the derived formulas for the transmission properties of 1-D MPSPCs. In general, the large incidence angles are decisive for the formation of ultra-wide PBGs but have the disadvantage of a reduction in the number of PBGs.

The variation of the reflection spectra of  $v_c$  transformed from  $0.1v_0$  ( $v_c = 0.00001\omega_p$ ) to  $10v_0$  ( $v_c = 0.001\omega_p$ ) is offered in Figure 10. The initial parameters are stipulated as follows:  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_P = \omega_0$ ,  $\omega_c = 0$ , and  $\alpha = 0$ . It can be roughly inferred from the figure that the change in  $v_c$  has an extraordinarily slight effect on the reflectance. To further investigate the effects of  $v_c$  on 1-D MPSPCs,  $v_c = 0.0001 \omega_p$ (in Figures 11(a) and 12(a)),  $v_c = 0.00001\omega_p$  (in Figures 11(b) and 12(b)) and  $v_c = 0.001\omega_p$ (in Figures 11(c) and 12(c)) are taken to compare the difference of reflectance spectra and dispersion curves among the three.

Combining Figures 11 and 12, it can be concluded that the PBGs arise at  $\omega/\omega_0 = 2.02$ , 2.60, and 2.87, and the transformation of  $v_c$  has almost no effect on the positions. However, the data comparison reveals that the increase of  $v_c$  has a trivial effect on the amplitude of the non-banned region. It is primarily shown that the larger the  $v_c$  is, the mild amplification of reflectance at a specific frequency in the non-banned region is observed. It makes sense to notice this property is thoroughly similar to that of 1-D PPC, which also indicates that it is unrealistic to try to obtain the change in positions and bandwidths of PBGs by adjusting  $v_c$ . Equally, the reflection spectra coincide well with the dispersion relation curves.

However, since the 1-D SPCs are analogous to the 1-D PPCs, differing only in geometric configuration and incident waveform, the superiority of the 1-D SPCs can be visualized



**Figure 6.** The influences of  $\omega_c$  on dispersion relations if  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ , (a)  $\omega_c = 0$ , (b)  $\omega_c = 0.8\omega_p$ , and (c)  $\omega_c = \omega_p$ .



**Figure 7.** The reflection pattern under varying  $\alpha$  with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $v_c = 0.0001\omega_p$ .

by comparing the two. A comparison of Figures 13 and 14 reveals that the reflection spectra of the 1-D MPSPCs are extremely analogous to the 1-D PPCs, indicating that the 1-D MPSPCs have the same preeminent PBG properties as 1-D PPCs. However, unlike the 1-D PPCs, the 1-D MPSPCs also have absorption properties. While the absorption of the 1-D PPCs is extremely weak with identical structural parameters, the 1-D MPSPCs have conspicuous absorption properties, and they can be further optimized by adjusting the incidence angle and reducing the initial radius. Therefore, it is reasonable to speculate that 1-D MPSPCs not only have splendid PBG properties but also have absorption characteristics that 1-D PPCs do not possess, which are particularly consequential for applications.



**Figure 8.** The reflection spectra in case of  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $v_c = 0.0001\omega_p$  when (a)  $\alpha = 0^\circ$ , (b)  $\alpha = 30^\circ$ , (c)  $\alpha = 45^\circ$ , and (d)  $\alpha = 60^\circ$ .



**Figure 9.** The dispersion relation curves with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $v_c = 0.0001\omega_p$  when (a)  $\alpha = 0^\circ$ , (b)  $\alpha = 30^\circ$ , (c)  $\alpha = 45^\circ$ , and (d)  $\alpha = 60^\circ$ .



**Figure 10.** Change in reflection spectra of  $v_c$  rising from 0.1 to 10 defining  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $\alpha = 0$  ( $v_0 = 0.0001\omega_p$ ).



**Figure 11.** The chart regarding the influence of  $v_c$  on reflection spectra when  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $\alpha = 0$ , (a) $v_c = 0.0001\omega_p$ , (b)  $v_c = 0.0001\omega_p$ , and (c)  $v_c = 0.001\omega_p$ .



**Figure 12.** The chart regarding the influence of  $v_c$  on dispersion relation when  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $\omega_c = 0$ , and  $\alpha = 0$ , (a) $v_c = 0.0001\omega_p$ , (b)  $v_c = 0.0001\omega_p$ , and (c)  $v_c = 0.001\omega_p$ .



**Figure 13.** The reflection, transmission and absorption spectra of 1-D PPCs of TM waves with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ .



**Figure 14.** The reflection, transmission and absorption spectra of 1-D MPSPCs of TM waves in the case of  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ .

By comparing Figures 14 and 15, we can easily find that the variations of the dissipation factor of dielectric K have no effect on the position and number of PBGs, and the two plots differ only in magnitude. In other words, the loss of energy only has an effect on the distribution of energy and has no bearing on the forbidden band peculiarities.

From Figure 16, the modification of  $\omega_p$  has a considerable impact on the PBGs. The individual parameters are set as follows:  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $v_c = 0.0001\omega_p$ ,  $\omega_c = 0$ , and  $\alpha = 0$ . When  $\omega_p$  is less than  $0.6\omega_0$ , the tunability is pretty weak and the change of PBGs is extremely trifling. After  $\omega_p$  enlarges to 0.6, the reflection spectrum is distinctly different from before, and the two-square marked areas in Figure 16 indicate the regions where the PBGs transform most dramatically, with alterations in the frequency range, bandwidth, and amount.

The reflection spectra and dispersion relationship curves of the proposed 1-D MPSPCs with  $\omega_p$  are provided in Figures 17 and 18. The other parameters are configured as  $r_0 = 20d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ ,  $\omega_c = 0$ , and  $\alpha = 0$ . First of all, let  $\omega_p = 0.6\omega_0$  and the PBGs can be expected at  $\omega/\omega_0 = 1.97$ , 2.54, and 2.81. The bandwidths of the PBGs are extraordinarily well-proportioned and the maximum bandwidth is  $0.14\omega_0$ . Following the line of variables discussion, increasing  $\omega_p$  to  $0.8\omega_0$ , the positions of the PBGs shifted slightly, varying to be at  $1.98\omega_0$ ,  $2.56\omega_0$ , and  $2.82\omega_0$ , and the frequency scale expanded marginally. Continuing to increase  $\omega_p$  to  $\omega_0$ , it is evidenced that PBGs show a comparable trend. It is worth mentioning that the number of PBGs aggrandizes for  $\omega_p = 1.2\omega_0$ , with PBGs at  $2.01\omega_0$ ,  $2.36\omega_0$ ,  $2.63\omega_0$ , and  $2.85\omega_0$ . The PBGs located at  $2.01\omega_0$  and  $2.85\omega_0$  do not change voluminously in terms of bandwidths, but the sharp decline in the bandwidth at  $2.63\omega_0$  is presumed to be



**Figure 15.** The reflection, transmission and absorption spectra of 1-D MPSPCs of TM waves with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ , dissipation factor of dielectric K tan $\sigma_e = 0.001$ , and  $\alpha = 0$ .



**Figure 16.** The graph of reflection with altering  $\omega_p$  defining  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_c = 0$ ,  $v_c = 0.0001\omega_p$  and  $\alpha = 0$ .

caused by the formation of a new PBG at  $2.36\omega_0$ .  $\omega_p$  is closely correlated with the electron density in the plasma layer, and an enlargement in the electron density leads to a rise in the energy consumption of the electromagnetic waves and thus to a phanerous variation



**Figure 17.** The reflection spectra about the influence of  $\omega_p$  with  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_c = 0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ , (a)  $\omega_p = 0.6 \omega_0$ , (b)  $\omega_p = 0.8\omega_0$ , (c)  $\omega_p = \omega_0$ , and (d)  $\omega_p = 1.2\omega_0$ .

in the transmission properties. It is necessary to complement that for the convenience of mathematical calculations, the Drude model is adopted with respect to the plasma [41]. The plasma frequency, as one of the crucial parameters of the plasma, is intimately intertwined with both density and temperature and is capable of reflecting the effects of density and temperature on 1-D MFSPCs transmission properties. Therefore, the temperature is not discussed separately here.

The medium thickness is a momentous factor affecting the PBGs. Also increasing the plasma thickness  $d_p$ , it can be seen in Figures 19(b) and 20(b) that PBGs are distributed at  $\omega/\omega_0 = 1.95$ , 2.21, and 2.50. In contrast, only two PBGs occur in the frequency range of  $1.9\omega_0-3.1\omega_0$ , located at  $\omega/\omega_0 = 2.08$  and 2.73 as shown in Figures 19(c) and 20(c). The diminution of  $d_p$  also has an effect on the number of PBGs, with  $d_A = 0.2d$ ,  $d_B = 0.45d$ , and  $d_P = 0.35d$ , a total of four PBGs are observed at  $1.91\omega_0$ ,  $2.12\omega_0$ ,  $2.40\omega_0$ , and  $2.70\omega_0$ , but the bandwidths of PBGs in this state are narrow. All things considered, the change in  $d_p$ has an enormous effect on PBGs and has prodigious control over their overall composition and frequency range. 1-D MPSPCs are sensitive to the adjustment of  $d_p$  by virtue of their geometrical properties. The high arbitrariness and adjustability of the PBGs distribution position are estimable for device design.

#### 4. Conclusion

Generally speaking, this paper presents the first theoretical derivation of the transmission properties of 1-D MPSPCs. Employing TMM, the expressions for the transmittance,

20



**Figure 18.** The dispersion relationship curves about the effects of  $\omega_p$  setting  $r_0 = 20d$ ,  $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ ,  $\omega_c = 0$ ,  $v_c = 0.0001\omega_p$ , and  $\alpha = 0$ , (a)  $\omega_p = 0.6\omega_0$ , (b)  $\omega_p = 0.8\omega_0$ , (c)  $\omega_p = \omega_0$ , and (d)  $\omega_p = 1.2\omega_0$ .

reflectance, and dispersion relations of 1-D MPSPCs under the TM waves are elicited. Meanwhile, the PBGs of 1-D MPSPCs are carefully analyzed by plotting the transmission spectra, reflection spectra, absorption spectra and dispersion relations curves. The reflection spectra of 1-D MPSPCs are in high agreement with the dispersion relation curves, which further proves the correctness of the theoretical derivation. The variables affecting 1-D MPSPCs are investigated in conjunction with the structure of the theoretical treatment. The effects of  $\omega_{cr}$  $\alpha$ ,  $v_c$ ,  $\omega_p$ , structure, tan $\sigma_{e}$ , and  $d_p$  on the spectra and dispersion relation curves are analyzed, respectively. By varying the values of the variables, it turns out that the impact of  $v_c$  on the PBGs is extremely tiny. There is a correlation between  $\omega_p$  and  $\omega_c$ , and the influences on PBGs are quite akin, prevailingly in making the position of PBGs move toward high frequencies and accompanied by the expansion of PBGs bandwidth and the enhancement in the number of them. The alterations of the  $\alpha$  and medium thickness will not only affect the position of PBGs but also the number of PBGs. Increasing both the incidence angle and plasma thickness can reduce the number and increase the bandwidth of PBGs. Structure and tan $\sigma_{e}$  have implications for energy distribution but no control on PBGs. And the reduction of plasma thickness can result in an increment in the number of PBGs. The properties of 1-D MPSPCs are similar to that of 1-D PPCs, but 1-D MPSPCs are more sensitive to the change of variables and have momentous practical significance in tunable devices and curved reflectors.

#### **Disclosure statement**

No potential conflict of interest was reported by the author(s).



**Figure 19.** The picture concerning the influence of  $d_p$  on reflection spectra at  $r_0 = 20d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ ,  $\omega_c = 0$ , and  $\alpha = 0$ , (a) $d_A = 0.2d$ ,  $d_B = 0.35d$ ,  $d_P = 0.45d$ , (b)  $d_A = 0.15d$ ,  $d_B = 0.3d$ ,  $d_P = 0.55d$ , (c)  $d_A = 0.1d$ ,  $d_B = 0.25d$ ,  $d_P = 0.65d$ , and (d)  $d_A = 0.2d$ ,  $d_B = 0.45d$ ,  $d_P = 0.35d$ .



**Figure 20.** The picture concerning the effects of  $d_p$  on dispersion relations if  $r_0 = 20d$ ,  $\omega_p = \omega_0$ ,  $v_c = 0.0001\omega_p$ ,  $\omega_c = 0$ , and  $\alpha = 0$ , (a) $d_A = 0.2d$ ,  $d_B = 0.35d$ , and  $d_P = 0.45d$ , (b)  $d_A = 0.15d$ ,  $d_B = 0.3d$ , and  $d_P = 0.55d$ , (c)  $d_A = 0.1d$ ,  $d_B = 0.25d$ , and  $d_P = 0.65d$ , and (d)  $d_A = 0.2d$ ,  $d_B = 0.45d$ ,  $d_P = 0.35d$ .

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24 🕢 T.-Q. ZHU ET AL.

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